NUCLEAR THERMAL-HYDRAULIC FUNDAMENTALS

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POWER CYCLES

The analysis of Thermodynamic Cycles is based almost exclusively on applications of the first and second law of thermodynamics to a control volume, along with mass conservation principles. The derivation of these conservation equations is covered in most elementary thermodynamic texts and as such the equations are simply repeated here without proof. Generally, these analyses provide global mass and energy balances with no indication of component size. The inherent assumption, is that the component or device can be constructed to provide the required performance.

First Law of Thermodynamics

The First Law is an energy conservation equation applied to a control volume. A common form of the First Law is

$$\dot{Q}_{c.v.} + \sum_{in} \dot{m}_{in}\left(h_{in} + \frac{v_{in}^2}{2g_c} + \frac{g}{g_c} z_{in}\right) = \frac{dE_{c.v.}}{dt} + \sum_{exit} \dot{m}_{exit}\left(h_{exit} + \frac{v_{exit}^2}{2g_c} + \frac{g}{g_c} z_{exit}\right) + \dot{W}_{c.v.} \quad (1)$$

where:

$\dot{Q}_{c.v.}$ = Heat transfer rate into (+) or out of (-) the control volume

$\sum_{in} \dot{m}_{in}\left(h_{in} + \frac{v_{in}^2}{2g_c} + \frac{g}{g_c} z_{in}\right)$ = Rate of energy convected into the control volume

$\sum_{exit} \dot{m}_{exit}\left(h_{exit} + \frac{v_{exit}^2}{2g_c} + \frac{g}{g_c} z_{exit}\right)$ = Rate of energy convected out of the control volume

$\frac{dE_{c.v.}}{dt}$ = Time rate of change of energy within the control volume

$\dot{W}_{c.v.}$ = Rate of work done on (-) or by (+) the control volume.

Note, the sign convention for work and heat transfer rate is somewhat arbitrary and other sign conventions are equally valid.

Most cycle analyses are performed at steady state, such that

$$\frac{dE_{c.v.}}{dt} = 0.$$

In addition, for most applications involving the analysis of power cycles, the kinetic and potential energy terms are negligible such that an effective working equation is

$$\dot{Q}_{c.v.} + \sum_{in} \dot{m}_{in} h_{in} = \sum_{exit} \dot{m}_{exit} h_{exit} + \dot{W}_{c.v.} \quad (2)$$

Mass Conservation

Conservation of mass for a control volume is given by

$$\frac{dM_{c.v.}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{exit} \dot{m}_{exit} \tag{3}$$

where $M_{c.v.}$ is the mass of control volume. Under steady state conditions

$$\frac{dM_{c.v.}}{dt} = 0$$

and

$$\sum_{in} \dot{m}_{in} = \sum_{exit} \dot{m}_{exit} \tag{4}$$

or the sum of the mass flow rates entering the control volume must equal the sum of the mass flow rates leaving the control volume.

Second Law of Thermodynamics for a Control Volume

The Second Law of Thermodynamics for a control volume is given by

$$\frac{dS_{c.v.}}{dt} + \sum_{exit} \dot{m}_{exit} s_{exit} \geq \sum_{in} \dot{m}_{in} s_{in} + \int_A \frac{\dot{Q}_{c.v.} / A}{T} dA \tag{5}$$

where s is entropy and A is the surface area of the control volume. The equality (=) refers to internally reversible processes, and the greater than sign (>) refers to internally irreversible processes. For steady state, reversible, adiabatic systems, Equation 5 reduces to

$$\sum_{exit} \dot{m}_{exit} s_{exit} = \sum_{in} \dot{m}_{in} s_{in} \tag{6}$$

which for single-inlet/single-outlet systems can be simplified to give

$$\dot{m} s_{exit} = \dot{m} s_{in} \tag{7}$$

or

$$s_{exit} = s_{in} \tag{8}$$

A constant entropy process is said to be **isentropic**.

Entropy is a fundamental thermodynamic property of the fluid and can be related to enthalpy through the expression

$$Tds = dh - \upsilon dP \tag{9}$$

where υ is specific volume and P is pressure. If we integrate between an arbitrary inlet and exit point along the flow stream,

$$\int_{inlet}^{exit} Tds = \int_{inlet}^{exit} dh - \int_{inlet}^{exit} \upsilon dP \qquad (10)$$

and assume a reversible, adiabatic (isentropic) process, i.e.

$$ds = 0 \Rightarrow \int_{inlet}^{exit} Tds = 0$$

then

$$\int_{inlet}^{exit} dh = \int_{inlet}^{exit} \upsilon dP \qquad (11)$$

or

$$h_{exit} - h_{inlet} = \int_{inlet}^{exit} \upsilon dP \qquad (12)$$

If we further assume the fluid density (or specific volume) is independent of pressure (incompressible), then υ is a constant and

$$h_{exit} - h_{inlet} = \upsilon(P_{exit} - P_{inlet}) \qquad (13)$$

Note, this expression is only valid for reversible, adiabatic processes involving fluids which can be approximated as incompressible. Physically, this situation holds for the ideal pumping of liquids.

Carnot Cycle

The Carnot Cycle is the most efficient cycle possible. It operates between a high temperature source and a low temperature sink and may be characterized by four basic processes:

1) A reversible-isothermal process in which heat is transferred to or from a high temperature reservoir.
2) A reversible-adiabatic process in which the temperature of the working fluid decreases from the high temperature to the low temperature.
3) A reversible-isothermal process in which heat is transferred to or from the low temperature reservoir.
4) A reversible-adiabatic process in which the temperature of the working fluid increases from the low temperature to the high temperature.

The efficiency of the Carnot Cycle is given by

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{Q}_h - \dot{Q}_l}{\dot{Q}_h} = 1 - \frac{T_l}{T_h} \tag{14}$$

where the temperatures are in absolute degrees. If we were to try and operate a nuclear power plant on this cycle, it could be approximated by the diagram in Figure 1.

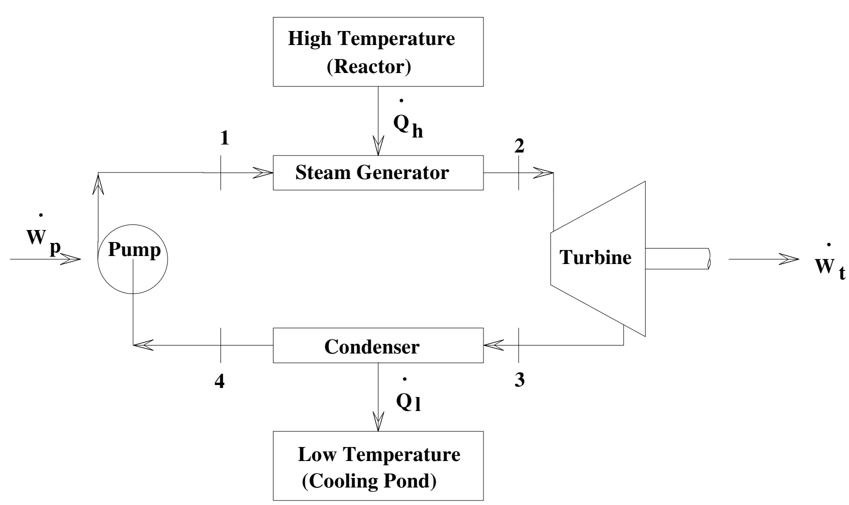


Figure 1: Carnot Cycle Representation of a Pressurized Water Reactor

We can represent the Carnot Cycle on a T-S diagram, where the saturated liquid and vapor lines for water have been added for illustration purposes.

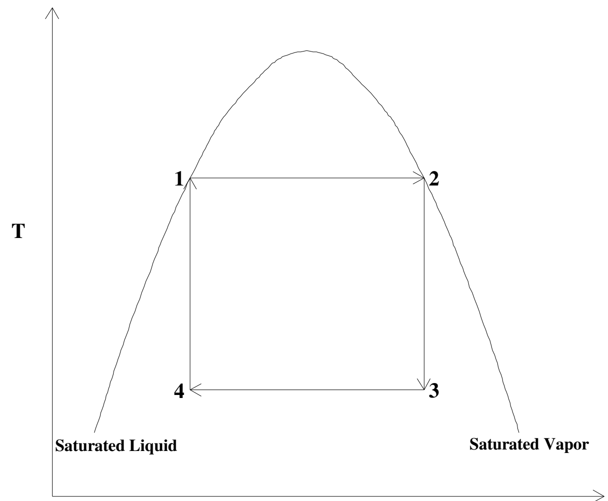


Figure 2: T-S Diagram for the Carnot Cycle

Note: The horizontal distance from point (3) to the saturated vapor line is a measure of the moisture contained within the steam at the turbine exhaust. Elevated moisture content results in increased erosion of turbine blades. Therefore, to reduce turbine wear it is desirable to maintain high steam quality. The cycle illustrated in Figure 2 implies a prohibitive moisture content at the turbine exhaust. To alleviate this problem requires that we either raise the condenser pressure (which would raise T_l and therefore lower the cycle efficiency), lower the steam generator pressure (which would lower T_h and therefore lower the cycle efficiency) and/or superheat the steam beyond the nominal state point at (2). The addition of superheat implies non isothermal heat addition and a deviation from the Carnot Cycle. In addition, the Carnot Cycle illustrated above would require that a two-phase mixture of liquid and vapor be pumped from state points (4) to (1). Designing and building pumps, which can operate under these conditions is usually impractical, and as a result most pumps are designed to operate with liquid water. This requires the steam to be fully condensed at (4). These deviations from the Carnot Cycle imply a new cycle for a practical steam power plant.

Rankine Cycle

The Rankine Cycle is the ideal cycle for a simple steam power plant. Given the power plant diagrammed below, the processes involved are:

1-2 Constant pressure heat transfer in the boiler
2-3 Reversible-adiabatic expansion in the turbine
3-4 Constant pressure heat transfer in the condenser
4-1 Reversible-adiabatic pumping to the boiler pressure

The line from 2 to 2′ allows for the option of superheating the steam in the boiler. The efficiency of the Rankine Cycle is most easily determined from the expression

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} \tag{15}$$

where $\dot{Q}_h$ is the heat addition in the boiler and $\dot{W}_{net}$ is the net rate of work done by all components in the system. Since the Rankine Cycle does not require constant temperature heat addition or rejection, in analyzing the Rankine Cycle, it is helpful to think of efficiency as depending on the average temperature at which heat is supplied and rejected. Any changes that increase the average temperature at which heat is added, or decrease the average temperature at which heat is rejected, will increase the Rankine Cycle efficiency. As both of these temperatures are dominated by the saturation temperatures (and therefore saturation pressures) in the boiler and condenser, the same efficiency arguments are true for increasing the boiler pressure and decreasing the condenser pressure.

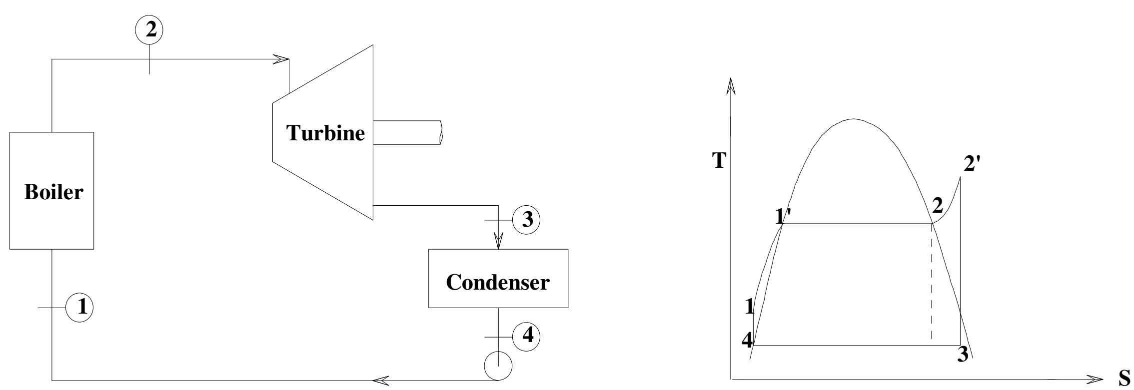


Figure 3: Ideal Rankine Cycle

Example:

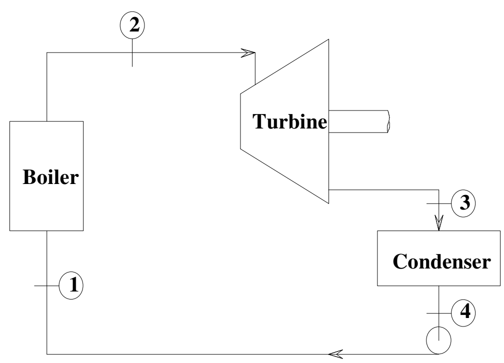
Determine the efficiency of a Rankine Cycle utilizing steam as the working fluid operating under the following conditions

Boiler Pressure = 900 psia
Boiler Superheat = 35 F
Condenser Pressure = 1 psia

Since the only work done in this cycle is by the turbine and condensate pump, the efficiency is given by

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_t/\dot{m} + \dot{W}_p/\dot{m}}{\dot{Q}_h/\dot{m}}$$

where we have normalized the work and heat transfer rates by the total system mass flow rate $\dot{m}$.



Turbine

From the steam tables:

h_2 = 1230.79 Btu/lbm
T_2 = 566.95 F
s_2 = 1.4371 Btu/R-lbm

In the absence of kinetic and potential energy terms, the First Law applied to the turbine is

$$\dot{m}h_2 = \dot{m}h_3 + \dot{W}_t$$

such that the turbine work per unit mass flow rate is given by

$$\dot{W}_t/\dot{m} = h_2 - h_3$$

The enthalpy at the turbine inlet is given by the boiler exit conditions. The enthalpy at the turbine exit is unknown. Since the condenser pressure alone is insufficient to specify the fluid's state at (3), additional information is required. This if obtained from the Second Law, which for a reversible-adiabatic turbine requires
$s_2 = s_3$

$$\therefore\ s_3 = 1.4371 \text{ Btu/R-lbm}$$

To determine the turbine exit conditions given the turbine exhaust pressure and entropy, note:

$$s_3 = \left(s_f + x s_{fg}\right)_3$$

$$\therefore x_3 = \left.\frac{s_3 - s_f}{s_{fg}}\right|_{P_3 = 1\ psia}$$

At 1 psia, s_f = .1326 Btu/R-lbm and s_{fg} = 1.8455 Btu/R-lbm giving

$$x_3 = \frac{1.4371 - 0.1326}{1.8455} = 0.7069.$$

The enthalpy at the turbine exhaust is then given by

$$h_3 = \left(h_f + x h_{fg}\right)_3$$

For h_f = 69.73 Btu/lbm and h_{fg} = 1036.1 Btu/lbm at 1 psia

$$h_3 = 69.73 + (0.7069)(1036.1) = 802.1 \text{ Btu/lbm.}$$

The turbine work per unit mass flow rate is then

$$\dot{W}_t / \dot{m} = h_2 - h_3 = 1230.79 - 802.1 = 428.69 \quad \text{Btu/lbm}$$

Pump

In the absence of kinetic and potential energy terms, the First Law applied to the condensate pump gives

$$\dot{m} h_4 = \dot{m} h_1 + \dot{W}_p$$

such that the pump work per unit mass flow rate is

$$-\dot{W}_p / \dot{m} = h_1 - h_4$$

For reversible-adiabatic processes where the density of the working fluid is approximately incompressible

$$h_1 = h_4 + \upsilon (P_1 - P_4)$$

such that

$$-\dot{W}_p / \dot{m} = \upsilon (P_1 - P_4)$$

For this example:

P_4 = 1 psia
P_1= 900 psia
υ = 0.0161 ft^3/lbm (evaluated at the pump inlet conditions)

Note: $\upsilon(P_e - P_i) = (0.0161)(900 - 1)(144) = 2084.24 \dfrac{\text{ft - lbf}}{\text{lbm}}$

To resolve the unit inconsistency, divide by 778 ft-lbf/Btu

$$-\dot{W}_p/\dot{m} = h_1 - h_4 = 2084.24 / 778 = 2.67 \text{ Btu/lbm}$$

For the ideal Rankine Cycle, the liquid leaving the condenser is saturated at the turbine exhaust pressure, such that

$$h_4 = h_f \text{ at 1 psia} = 69.73 \text{ Btu/lbm}$$

$$h_1 = 69.73 + 2.67 = 72.41 \text{ Btu/lbm.}$$

Boiler

The First Law applied to the boiler gives

$$\dot{Q} + \dot{m}h_1 = \dot{m}h_2 \Rightarrow \dot{Q}/\dot{m} = h_2 - h_1$$

$$\dot{Q}/\dot{m} = h_2 - h_1 = 1230.79 - 72.41 = 1158.38 \text{ Btu/lbm}$$

Cycle Efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_t/\dot{m} + \dot{W}_p/\dot{m}}{\dot{Q}_h/\dot{m}} = \frac{428.69 - 2.67}{1158.38} = 36.78\%$$

Note, the cycle efficiency is independent of the magnitude of the mass flow rate. While not required for the efficiency calculation, it is also of interest to examine the heat transfer rate across the condenser

Condenser

$$\dot{Q}_{cond}/\dot{m} = h_4 - h_3 = 69.73 - 802.1 = -732.37 \text{ Btu/lbm}$$

The (-) sign indicates heat transfer out of the control volume (cooling tower, cooling pond, etc.). Of the 1158.38 Btu/lbm added in the boiler, only 428.69 Btu/lbm, or a little over 1/3 was extracted as work in the turbine. The remainder is dumped across the condenser and discharged to the environment. It is also of interest to compare the pump work to the turbine work. For Rankine type steam cycles, the pump work is generally much less than the turbine work, such that it is a reasonable approximation to relate the heat input in the boiler to the turbine (or generator) output by

$$\dot{Q}_h \times \eta \cong \dot{W}_t .$$

Effect of Pressure and Temperature on the Rankine Cycle

1) Turbine Exhaust (Condenser) Pressure and Temperature

   Decreasing the turbine exhaust pressure decreases the saturation temperature in the condenser and therefore the average temperature at which heat is rejected. This implies an increase in the cycle efficiency. However, examination of the T-S diagram for the Rankine Cycle indicates, that for a given boiler pressure, lowering the condenser pressure results in an increase in the moisture content at the turbine exhaust and enhanced erosion of the turbine blades.

2) Boiler Pressure

   Increasing boiler pressure results in an increased saturation temperature in the boiler and therefore an increase in the temperature at which heat is added. This implies an increase in cycle efficiency. However, as in decreasing the condenser pressure, increasing boiler pressure for a given condenser pressure results in an increase in moisture content at the turbine exhaust.

3) Boiler Superheat

   Superheating in the boiler increases the average temperature at which heat is added and therefore would be expected to increase the cycle efficiency. Examination of the T-S diagram for the Rankine Cycle shows that superheating also results in a **decrease** in moisture content at the turbine exhaust.

Figures 4 and 5 below illustrate the effect of boiler pressure on a simple Rankine Cycle in the absence of superheat. Clearly, the moisture content at the turbine exhaust quickly becomes excessive. Figure 6 gives the temperature the steam must be superheated to at the corresponding boiler pressure to maintain a moisture content of 10%.

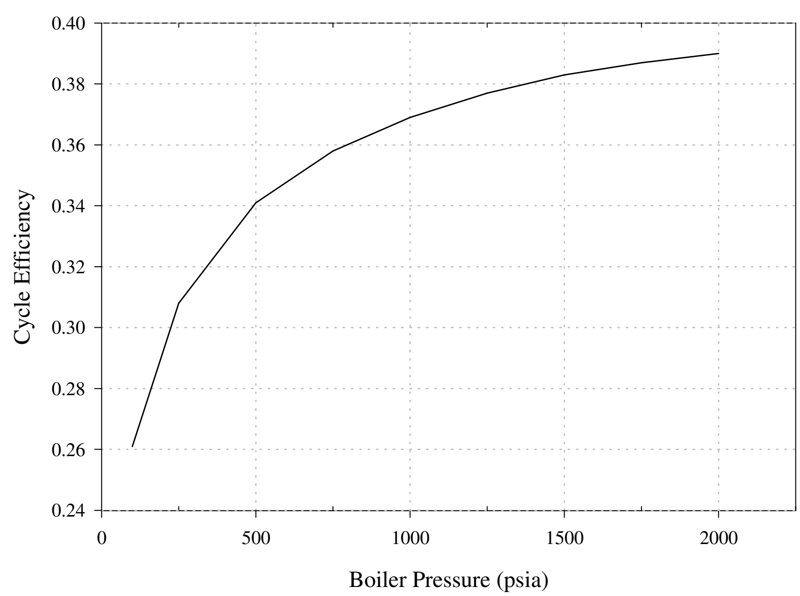


Figure 4: Cycle Efficiency Versus Boiler Pressure

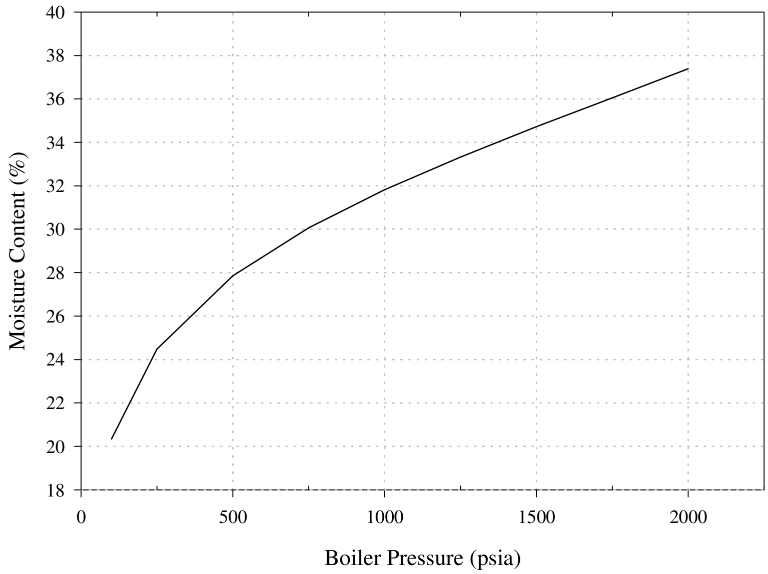


Figure 5: Moisture Content at the Turbine Exhaust Versus Boiler Pressure

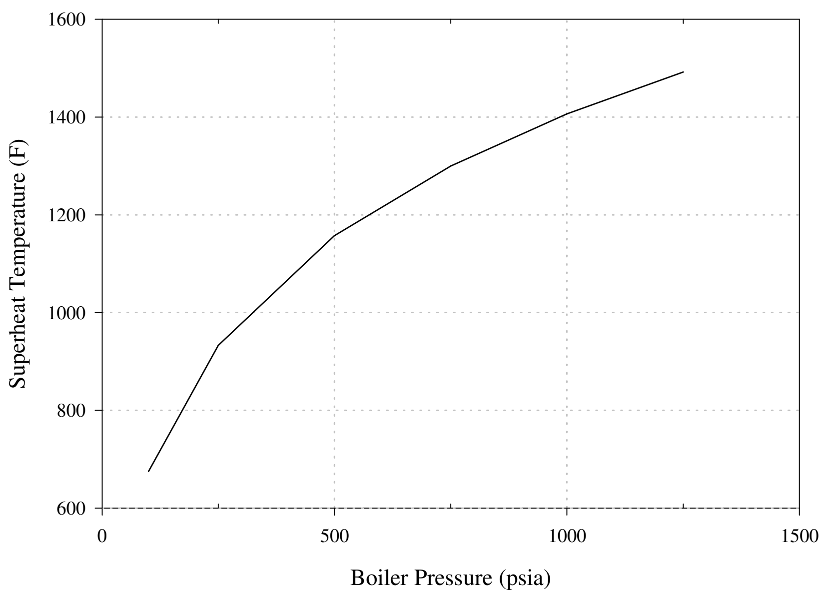


Figure 6: Steam Temperature Necessary to Maintain 10% Moisture Content

Reheat Cycle

It has been shown, that increasing the boiler pressure leads to increases in cycle efficiency, but at the expense of a higher moisture content at the turbine exhaust. It has also been shown that superheating is effective in reducing the moisture content at higher boiler pressures, but can lead to excessive steam temperatures. The Reheat Cycle is a way to take advantage of the higher boiler pressures while maintaining high steam quality at the turbine exhaust and moderate steam temperatures. Classically, this is accomplished by expanding the steam to some intermediate pressure in the turbine and then reheating it in the boiler before expanding through the final turbine stages to the exhaust pressure. This cycle is diagrammed in Figure 7 below.

The efficiency of the Reheat Cycle is in general only slightly (though not insignificantly) higher than the corresponding Rankine Cycle. The chief advantage is the decreased moisture content at the turbine exhaust. If the component materials could withstand superheating to point 2' on the T-S diagram, the Rankine Cycle would have the same turbine exhaust quality and a higher overall cycle efficiency and the Reheat Cycle would not be necessary.

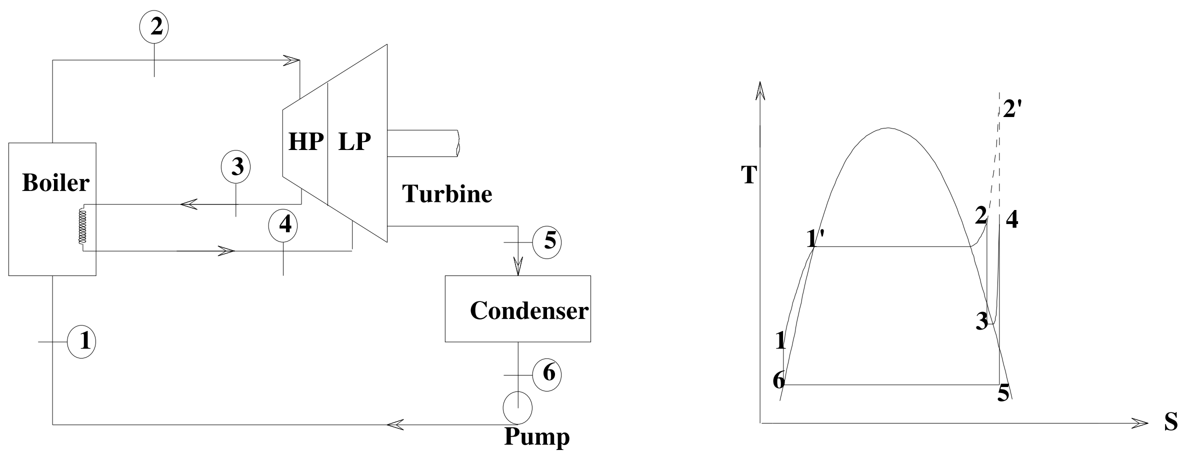


Figure 7: Classic Reheat Cycle

Example:

Boiler Pressure = 900 psia
Boiler Superheat = 35 F
Reheat Pressure = 200 psia
Condenser Pressure = 1 psia

Determine the cycle efficiency.

High Pressure Turbine

h_2 = 1230.79 Btu/lbm
T_2 = 566.95 F
s_2 = 1.4371 Btu/R-lbm

For a reversible-adiabatic turbine $s_2 = s_3$

$$\therefore\ s_3 = 1.4371 \text{ Btu/R-lbm}$$

$$s_3 = \left(s_f + x s_{fg}\right)_3 \Rightarrow x_3 = \left.\frac{s_3 - s_f}{s_{fg}}\right|_{P_3}$$

At 200 psia, s_f = 0.5438 Btu/R-lbm and s_{fg} = 1.0016 Btu/R-lbm giving

$$x_3 = \frac{1.4371 - 0.5438}{1.0016} = 0.8919 .$$

$$h_3 = \left(h_f + x h_{fg}\right)_3$$

At 200 psia, h_f = 355.3 Btu/lbm and h_{fg} = 842.8 Btu/lbm giving

$$h_3 = 355.5 + (0.8919)(842.8) = 1107.19 \text{ Btu/lbm.}$$

The turbine work per unit mass flow rate in the **high pressure stage** of the turbine is then

$$\dot{W}_{hp} \big/ \dot{m} = h_2 - h_3 = 1230.79 - 1107.19 = 123.6 \quad \text{Btu/lbm}$$

Low Pressure Turbine

For the ideal reheat cycle, it is normally assumed that $T_4 = T_2$.

h_4 = 1305.15 Btu/lbm
T_4 = 566.95 F
s_4 = 1.6604 Btu/R-lbm (note, $s_4 \neq s_2$)

$$s_5 = s_4 = 1.6604 \text{ Btu/R-lbm}$$

$$s_5 = \left(s_f + x s_{fg}\right)_5 \Rightarrow x_5 = \left.\frac{s_5 - s_f}{s_{fg}}\right|_{P_5}$$

At 1 psia, s_f = 0.1326 Btu/R-lbm and s_{fg} = 1.8455 Btu/R-lbm giving

$$x_5 = \frac{1.6604 - 0.1326}{1.8455} = 0.8279 .$$

$$h_5 = \left(h_f + x h_{fg}\right)_5$$

For h_f = 69.73 Btu/lbm and h_{fg} = 1036.1 Btu/lbm at 1 psia

$$h_5 = 69.73 + (0.8279)(1036.1) = 927.52 \text{ Btu/lbm.}$$

The turbine work per unit mass flow rate in the **low pressure stage** of the turbine is then

$$\dot{W}_{lp} \big/ \dot{m} = h_4 - h_5 = 1305.15 - 927.52 = 377.63 \quad \text{Btu/lbm}$$

Pump

The pump work is determined from

$$-\dot{W}_p/\dot{m} = h_1 - h_6$$

For a reversible-adiabatic pump

$$h_e = h_i + \upsilon(P_e - P_i)$$

P_i = 1 psia
P_e = 900 psia
υ = 0.0161 ft^3/lbm

$$\upsilon(P_e - P_i) = (0.0161)(900 - 1)(144/778) = 2.67$$

$$-\dot{W}_p/\dot{m} = 2.67 \text{ Btu/lbm}$$

$$h_1 = 69.73 + 2.67 = 72.41 \text{ Btu/lbm.}$$

Boiler

The heat input from the boiler must now include the reheat stage such that

$$\begin{aligned}\dot{Q}/\dot{m} &= h_2 - h_1 + h_4 - h_3 \\ &= 1230.79 - 72.41 + 1305.15 - 1107.19 \\ &= 1356.34 \text{ Btu/lbm}\end{aligned}$$

Cycle Efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_{hp}/\dot{m} + \dot{W}_{lp}/\dot{m} + \dot{W}_p/\dot{m}}{\dot{Q}_h/\dot{m}} = \frac{123.6 + 377.63 - 2.67}{1356.34} = 36.76\%$$

An alternative to reheating in the boiler commonly employed in nuclear power systems, is to extract some small amount of steam prior to entering the high pressure turbine, and use this steam in an external reheater prior to entering the low pressure turbine as illustrated below. Note, that the maximum reheat temperature in this design is the steam temperature leaving the boiler. It can be shown, that if implemented properly, this cycle has the same efficiency as that of the classic reheat cycle.

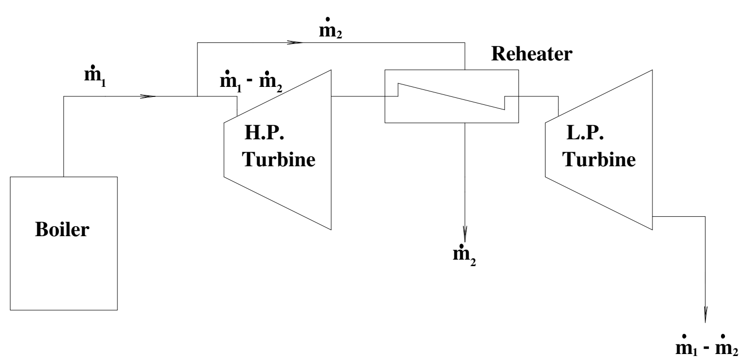


Figure 8: Alternate Reheat Cycle

<u>Regenerative Cycle</u>

Consider the Rankine Cycle without superheat.

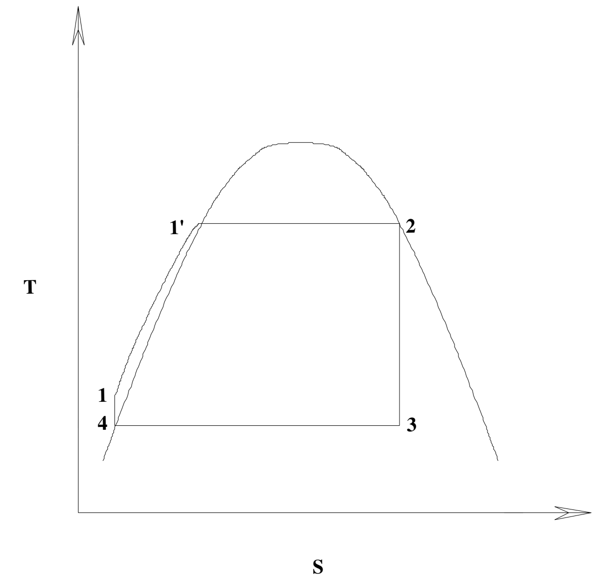


Figure 9: T-S Diagram for a Rankine Cycle Without Superheat

Between 1 and 1', the working fluid is being heated to the saturation point. The corresponding average temperature of the fluid is much lower than in the vaporization process from 1' to 2. Obviously, if the working fluid could enter the boiler closer to the saturation point, i.e. between 1 and 1', while maintaining the same condenser pressure the cycle efficiency would be improved. The Regenerative Cycle accomplishes this by extracting a relatively small amount of steam from the turbine after it has partially expanded and using it to heat the feedwater in *feedwater heaters*. The cycle diagram is illustrated below.

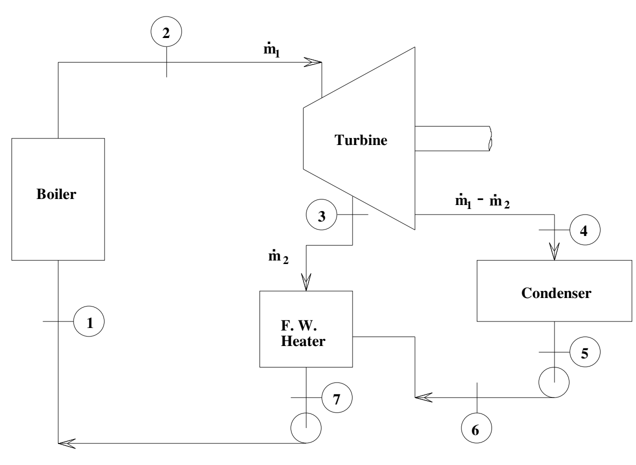


Figure 10: Simple Regenerative Cycle

The physical processes involved are:

a) Steam enters the turbine at (2) and expands to some intermediate state designated as point (3).

b) At (3), some of the steam is extracted and enters the feedwater heater.

c) The remaining steam is expanded to the condenser pressure at (4) and condensed in the condenser.

d) The condensate is pumped to the feedwater heater where it mixes with the extraction steam from the turbine. The amount of steam extracted at (3) is just enough to heat the entering condensate to the saturation point at (7).

e) The saturated liquid leaving the feedwater heater is pumped to the boiler pressure at (1).

The associated T-S diagram is given below.

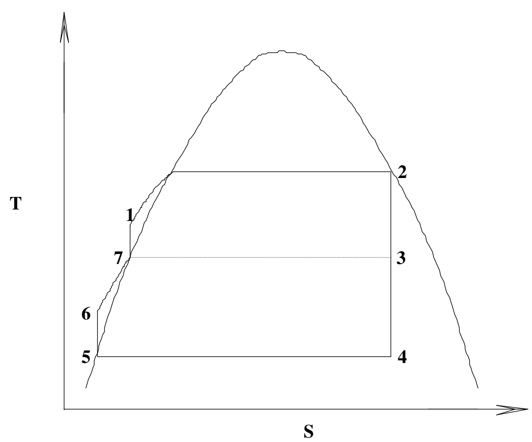


Figure 11: T-S Diagram for a Simple Regenerative Cycle

Example:

Boiler Pressure = 900 psia
Boiler Superheat = 35 F
Tap Pressure = 200 psia
Condenser Pressure = 1 psia

Determine the cycle efficiency.

High Pressure Turbine

h_2 = 1230.79 Btu/lbm
T_2 = 566.95 F
s_2 = 1.4371 Btu/R-lbm

For a reversible-adiabatic turbine $s_2 = s_3 = s_4$ = 1.4371 Btu/R-lbm

$$s_3 = \left(s_f + x s_{fg}\right)_3 \Rightarrow x_3 = \left.\frac{s_3 - s_f}{s_{fg}}\right|_{P_3}$$

At 200 psia, s_f = 0.5438 Btu/R-lbm and s_{fg} = 1.0016 Btu/R-lbm giving

$$x_3 = \frac{1.4371 - 0.5438}{1.0016} = 0.8919 .$$

$$h_3 = \left(h_f + x h_{fg}\right)_3$$

At 200 psia, h_f = 355.3 Btu/lbm and h_{fg} = 842.8 Btu/lbm giving

$$h_3 = 355.5 + (0.8919)(842.8) = 1107.19 \text{ Btu/lbm.}$$

$$s_4 = \left(s_f + x s_{fg}\right)_4 \Rightarrow x_4 = \left.\frac{s_4 - s_f}{s_{fg}}\right|_{P_4}$$

At 1 psia, s_f = 0.1326 Btu/R-lbm and s_{fg} = 1.8455 Btu/R-lbm giving

$$x_4 = \frac{1.4371 - 0.1326}{1.8455} = 0.7069 .$$

$$h_4 = \left(h_f + x h_{fg}\right)_4$$

For h_f = 69.73 Btu/lbm and h_{fg} = 1036.1 Btu/lbm at 1 psia

$$h_4 = 69.73 + (0.7069)(1036.1) = 802.1 \text{ Btu/lbm.}$$

<u>Condensate Pump</u>

P_5 = 1 psia
P_6 = 200 psia
υ = 0.0161 ft^3/lbm

$$\begin{aligned} h_6 &= h_5 + \upsilon(P_6 - P_5) \\ &= 69.73 + (0.0161)(200 - 1)(144/778) \\ &= 69.73 + 0.593 \\ &= 70.32 \text{ Btu/lbm} \end{aligned}$$

<u>Feedwater Heater</u>

Apply the First Law to the feedwater heater.

$$\dot{m}_2 h_3 + (\dot{m}_1 - \dot{m}_2) h_6 = \dot{m}_1 h_7$$

Neither mass flow rate is known, however as we have seen in previous examples, efficiency is independent of the magnitude of the mass flow rate. We can then solve for the relative mass flow rate

$$\frac{\dot{m}_2}{\dot{m}_1} = \frac{h_7 - h_6}{h_3 - h_6}$$

Note: $h_7 = h_f$ at 200 psia = 355.5 Btu/lbm

$$\frac{\dot{m}_2}{\dot{m}_1} = \frac{355.5 - 70.32}{1107.19 - 70.32} = 0.275$$

Feed Pump

P_1 = 900 psia
P_7 = 200 psia
υ = 0.0184 ft^3/lbm

$$\begin{aligned} h_1 &= h_7 + \upsilon(P_1 - P_7) \\ &= 355.5 + (0.0184)(900 - 200)(144/778) \\ &= 355.5 + 2.38 \\ &= 357.88 \text{ Btu/lbm} \end{aligned}$$

Boiler

$$\begin{aligned} \dot{Q}/\dot{m}_1 &= h_2 - h_1 \\ &= 1230.79 - 357.88 \\ &= 872.91 \text{ Btu/lbm} \end{aligned}$$

Cycle Efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_t + \dot{W}_{cp} + \dot{W}_{fp}}{\dot{Q}_h}$$

Note: Since the mass flow rate is not uniform throughout the cycle, we must account for this is determining the individual works and heat transfer rates. We can still normalize each of these terms by the total system mass flow rate such that the efficiency is written

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_t / \dot{m}_1 + \dot{W}_{cp} / \dot{m}_1 + \dot{W}_{fp} / \dot{m}_1}{\dot{Q}_h / \dot{m}_1}$$

Turbine Work

$$\dot{m}_1 h_2 = \dot{m}_2 h_3 + (\dot{m}_1 - \dot{m}_2) h_4 + \dot{W}_t$$

$$\dot{W}_t / \dot{m}_1 = h_2 - (\dot{m}_2/\dot{m}_1) h_3 - (1 - \dot{m}_2/\dot{m}_1) h_4$$

$$\dot{W}_t / \dot{m}_1 = 1230.79 - (0.275)(1170.19) - (0.725)(802.1) = 344.79 \text{ Btu/lbm}$$

Pump Work

$$-\dot{W}_{cp} = (\dot{m}_1 - \dot{m}_2)(h_6 - h_5)$$

$$\begin{aligned} -\dot{W}_{cp} / \dot{m}_1 &= (1 - \dot{m}_2/\dot{m}_1)(h_6 - h_5) \\ &= (0.725)(.593) \\ &= 0.43 \text{ Btu/lbm} \end{aligned}$$

$$-\dot{W}_{fp} = \dot{m}_1 (h_1 - h_7)$$

$$-\dot{W}_{fp}\Big/\dot{m}_1 = (h_1 - h_7)$$
$$= 2.38 \text{ Btu/lbm}$$

$$\eta = \frac{344.79 - 0.43 - 2.38}{872.91} = 39.18\%$$

This efficiency should be compared to the 36.78 % efficiency of the Rankine operating between the same boiler and condenser conditions.

The type of heater discussed in the previous example is called an *open* or *deaerating feedwater heater* as there is no physical separation of the inlet streams. Another common type of feedwater heater separates the extraction steam from the feedwater via tubes as illustrated below. This type of heater is referred to as a *closed feedwater heater*. Heat transfer is accomplished through condensation on the tube walls. The condensate may be pumped into the feedwater line or allowed to drain to a lower pressure heater or the condenser.

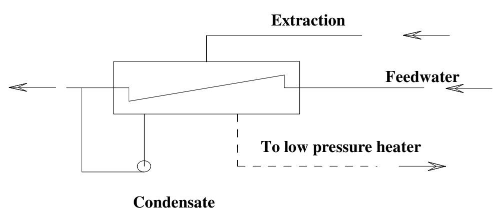


Figure 12: Closed Feedwater Heater

Open feedwater heaters have the advantage of cheap cost and better heat transfer characteristics than closed heaters, however they require a pump for each heater. Closed feedwater heaters can operate with one pump per several heaters. Most power plants utilize several feedwater heaters, with the final number limited by economic considerations.

The efficiency of the Regenerative Cycle is a function of the turbine tap pressure at which feedwater heating is to be performed. In general, optimization techniques are required to determine the tap pressures and feed temperatures which maximize the efficiency of any given cycle. However, a reasonable approximation to the optimum configuration can be obtained when the outlet temperature from each heater is equally spaced between the condenser temperature and the boiler saturation temperature. For a single heater then, the optimum heater outlet temperature is midway between the boiler and condenser saturation temperatures. This is illustrated in Figure 13 below for the Regenerative Cycle in the previous example. Cycle efficiency is given as a function of the turbine tap pressure with the optimum approximately 80 psia. Since the fluid leaves the heater as a saturated liquid, the corresponding feed temperature is then approximately 312 F. For a boiler pressure of 900 psia (Tsat ≅ 532 F) and a condenser pressure of 1 psia (Tsat ≅ 102 F), the optimum feed temperature should be approximately 317 F which corresponds to a tap pressure of approximately 85 psia.

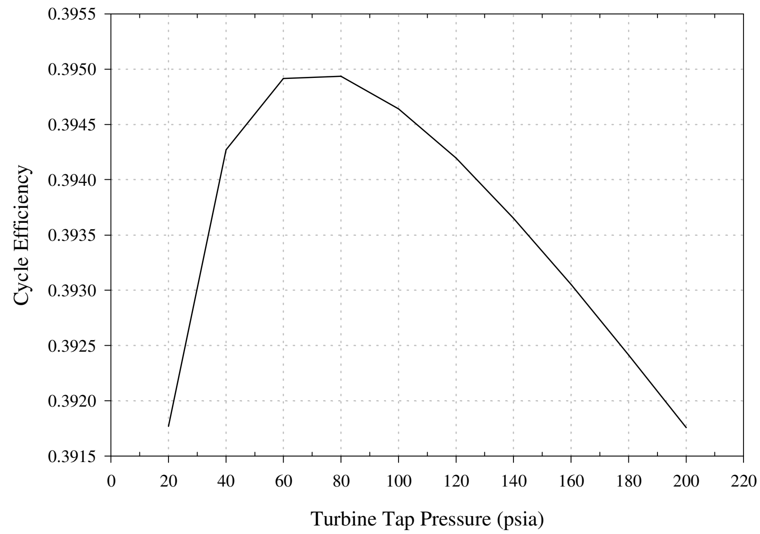


Figure 13: Regenerative Cycle Efficiency Versus Tap Pressure

Plants commonly incorporate a reheat stage with moisture separation in addition to the feedwater heaters as illustrated in Figures 14 and 15. The moisture separator acts to remove water droplets from the steam resulting in a higher quality at the separator exit and therefore more efficient reheating as this moisture does not have to be re-evaporated.

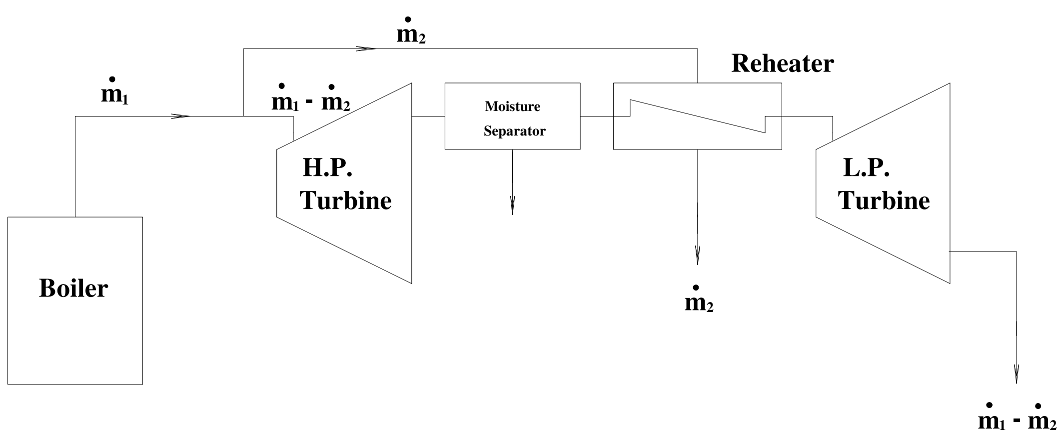


Figure 14: Moister Separator and Reheater

An ideal separator would result in complete removal of all moisture, such that the steam exits the separator as a saturated vapor as illustrated below. The extracted moisture can then be diverted to some other point in the cycle for feed water heating.

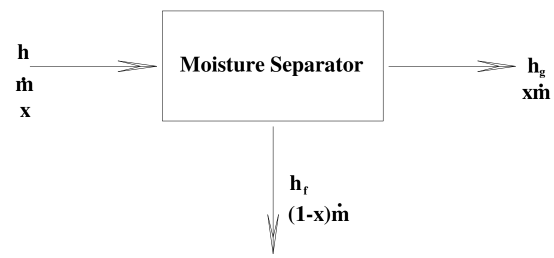


Figure 15: Ideal Moisture Separator

Deviations of Actual Cycles From Ideal Cycles

Actual cycle efficiencies differ from those computed for ideal cycles due to irreversible losses in piping and cycle components. Examples of losses which may affect the overall cycle efficiency include:

1) *Piping Losses*

Piping losses result primarily from heat loss to the environment as well as pressure loss due to friction.

2) *Turbine Losses*

Actual turbines are not reversible-adiabatic machines. The deviation of an actual turbine's performance from that of an ideal turbine is given in terms of a turbine efficiency. This efficiency is defined such that

$$\eta_t = \frac{\dot{W}_{ta}}{\dot{W}_{ts}}$$

where:

$\dot{W}_{ta}$ = Actual turbine work output
$\dot{W}_{ts}$ = Ideal (isentropic) turbine work output

3) *Pumping losses*

As in turbines, actual pumps are not reversible-adiabatic machines. Pump performance is also characterized by a pump efficiency defined such that

$$\eta_p = \frac{\dot{W}_{ps}}{\dot{W}_{pa}}$$

where:

$\dot{W}_{pa}$ = Actual work input of the pump
$\dot{W}_{ps}$ = Ideal (isentropic) work input of the pump

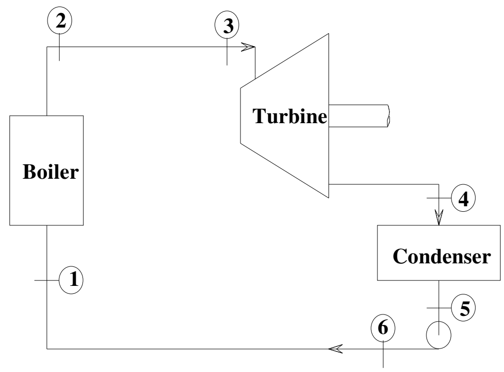
Note, in computing pump work, the isentropic work appears in the numerator.

4) *Condenser losses*

Condenser losses are usually minor. One such loss is that due to subcooling of the working fluid prior to return to the boiler.

Example:

Consider the simple Rankine Cycle illustrate below with the following state point conditions. For this example, assume a turbine efficiency of 85% and a pump efficiency of 82%.



Location	Pressure (psia)	Temperature (F)
1	960	90
2	900	566.75
3	890	550
4	1	
5	1	95
6	970	

Turbine

h_3 = 1217 Btu/lbm
s_3= 1.4248 Btu/R-lbm

(a) Ideal Turbine Work

$s_4 = s_3 = 1.4248$

$$s_4 = \left(s_f + xs_{fg}\right)\Big|_4 \Rightarrow x_{4s} = \frac{s_4 - s_f}{s_{fg}}\Bigg|_{P_4}$$

At P_4 = 1 psia

$s_f = 0.132$
$s_{fg} = 1.8455$

$$x_{4s} = \frac{1.4248 - .1326}{1.8455} = 0.7002$$

$$h_{4s} = \left(h_f + xh_{fg}\right)_4$$

At P_4 = 1 psia

$h_f = 69.73$
$h_{fg} = 1036.1$

$\therefore\ h_{4s} = 69.73 + (.7002)(1036.1) = 795.20$ Btu/lbm

$\dot{W}_{ts}/\dot{m} = h_3 - h_{4s} = 1217 - 795.2 = 421.8$ Btu/lbm

(b) Actual Work Output

$$\eta_t = \frac{\dot{W}_{ta}}{\dot{W}_{ts}} = \frac{\dot{m}(h_3 - h_{4a})}{\dot{W}_{ts}}$$

$$\therefore h_{4a} = h_3 - \eta_t \dot{W}_{ts}/\dot{m}$$

$$\frac{\dot{W}_{ta}}{\dot{m}} = \eta_t \frac{\dot{W}_{ts}}{\dot{m}} = (0.85)(421.8) = 358.53 \text{ Btu/lbm}$$

$$h_{4a} = 1217 - (0.85)(421.8) = 858.47 \text{ Btu/lbm}$$

Pump

(a) Ideal Pump Work

$$-w_{ps} = \upsilon(P_6 - P_5) = -\dot{W}_{ps}/\dot{m}$$
$$-w_{ps} = (0.0161)(970 - 1)(144/778) = 2.89 \text{ Btu/lbm}$$

(b) Actual Pump Work

$$-w_{pa} = -w_{ps}/\eta_p$$
$$-w_{pa} = 2.89/0.82 = 3.52 \text{ Btu/lbm}$$

Note:

$$h_{6a} = h_5 - w_{pa}$$
$$= 63.01 + 3.52$$
$$h_{6a} = 66.53 \text{ Btu/lbm}$$

Boiler

$$\dot{Q}/\dot{m} = h_2 - h_1$$

$h_1 = 58.02$ Btu/lbm $\qquad h_2 = 1230.79$ Btu/lbm

$$\dot{Q}/\dot{m} = 1230.79 - 58.02 = 1172.77 \text{ Btu/lbm}$$

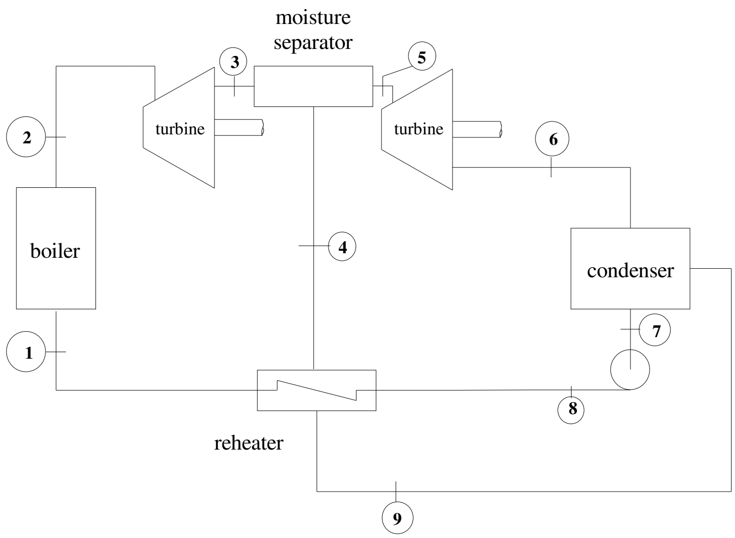
Efficiency

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_h} = \frac{\dot{W}_{ta}/\dot{m} + \dot{W}_{pa}/\dot{m}}{\dot{Q}_h/\dot{m}} = \frac{358.53 - 3.52}{1172.77} = 30.72\%$$

Example:

Saturated steam at 1000 psia enters a high pressure turbine with an efficiency of 85%. The steam expands to 100 psia where it enters a moisture separator and all entrained moisture is removed. Saturated steam at 100 psia is then expanded through a low pressure turbine of 85% efficiency to the condenser at 1 psia. Saturated liquid from the moisture separator is sent to a closed feedwater heater where it is cooled to 110 F before being sent to the condenser. For a pump efficiency of 85% calculate the overall plant efficiency.

SOLUTION



Turbine

$h_2 = h_g$ @ 1000 psia = 1192.9 Btu/lbm
$s_2 = s_g$ @ 1000 psia = 1.391

The enthalpy at the turbine exhaust and actual work done by the high pressure turbine is given by

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3_s}} = \frac{\dot{W}_{thpa}}{\dot{W}_{thps}}$$

such that

$$h_3 = h_2 - \eta_t (h_2 - h_{3_s})$$

To determine the quality and enthalpy at the high pressure turbine exhaust

$$x_{3_s} = \frac{s_2 - s_f}{s_{fg}} \text{ @ } P_3 = 100 \text{ psia}$$

$s_f = 0.4743$
$s_{fg} = 1.1284$

h_f = 298.5 Btu/lbm
h_{fg} = 888.6 Btu/lbm

$$x_{3_s} = \frac{1.391 - 0.4743}{1.1284} = 0.8124$$

$$h_{3_s} = h_f + x_{3_s} h_{fg} \ @ \ P_3 = 100 \, \text{psia}$$
$$h_{3_s} = 298.5 + (0.8124)(888.6)$$
$$h_{3_s} = 1020.4$$

$$h_3 = h_2 - \eta_t (h_2 - h_{3_s})$$
$$h_3 = 1192.9 - 0.85(1192.9 - 1020.4)$$
$$h_3 = 1046.3$$

$$w_{thp_a} = h_2 - h_3$$
$$w_{thp_a} = 1192.9 - 1046.3$$
$$w_{thp_a} = 146.6$$

The true quality at the turbine exhaust is then

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{1046.3 - 298.5}{888.6} = 0.8415$$

<u>Moisture Separator</u>

Assuming the moisture separator to be ideal

$$\dot{m}_2 = (1 - x_3)\dot{m}_1$$
$$\dot{m}_2 / \dot{m}_1 = 1 - x_3 = 1 - 0.8415 = 0.1585$$

<u>Low Pressure Turbine</u>

$h_5 = h_g$ @ P_3 = 100 psia
$s_5 = s_g$ @ P_3 = 100 psia
h_5 = 1187.2 Btu/lbm
s_5 = 1.6027

The enthalpy at the turbine exhaust and actual work done by the low pressure turbine is given by

$$\eta_t = \frac{h_5 - h_6}{h_5 - h_{6_s}} = \frac{\dot{W}_{tlpa}}{\dot{W}_{tlps}}$$

such that

$$h_6 = h_5 - \eta_t (h_5 - h_{6_s})$$

To determine the enthalpy at the low pressure turbine exhaust

$$x_{6_s} = \frac{s_5 - s_f}{s_{fg}} \text{ @ } P_6 = 1 \text{ psia}$$

$s_f = 0.1326$
$s_{fg} = 1.8455$

$h_f = 69.73$ Btu/lbm
$h_{fg} = 1036.1$ Btu/lbm

$$x_{6_s} = \frac{1.6027 - 0.1326}{1.8455} = 0.7966$$

$$h_{6_s} = h_f + x_{6_s} h_{fg} \text{ @ } P_6 = 1 \text{psia}$$
$$h_{6_s} = 69.73 + (0.7966)(1036.1)$$
$$h_{6_s} = 895.1$$

$$h_6 = h_5 - \eta_t (h_5 - h_{6_s})$$
$$h_6 = 1187.2 - 0.85(1187.2 - 895.1)$$
$$h_6 = 938.9$$

$$w_{tlp_a} = h_5 - h_6$$
$$w_{tlp_a} = 1187.2 - 938.9$$
$$w_{tlp_a} = 248.3$$

Condensate Pump

$$-w_{cp_s} = \upsilon(P_1 - P_7)$$
$$-w_{cp_s} = (0.0161)(1000 - 1)(144/778)$$
$$-w_{cp_s} = 2.98$$

$$w_{cp_a} = \frac{w_{cp_s}}{\eta_p}$$
$$w_{cp_a} = \frac{-2.98}{0.85} = -3.51 \text{ Btu/lbm}$$

$$h_8 = h_7 - w_{cp_a}$$
$$h_8 = 69.73 + 3.51$$
$$h_8 = 73.24$$

Reheater

Application of the first law to the reheater gives

$$\dot{m}_1 h_1 + \dot{m}_2 h_9 = \dot{m}_1 h_8 + \dot{m}_2 h_4$$

Since the mass flow rate ratios are known, we solve for the boiler inlet enthalpy

$h_4 = h_f$ @ P_3 = 100 psia
h_4 = 298.5 Btu/lbm
h_9 = 77.98 Btu/lbm

$$h_1 = h_8 + \left(\dot{m}_2 / \dot{m}_1\right)(h_4 - h_9)$$
$$h_1 = 73.24 + (0.1585)(298.5 - 77.98)$$
$$h_1 = 108.19$$

Boiler

$$q = h_2 - h_1$$
$$q = 1192.9 - 108.19$$
$$q = 1084.7$$

Cycle Efficiency

$$\eta = \frac{\dot{W}_{thp} + \dot{W}_{tlp} + \dot{W}_{cp}}{\dot{Q}} = \frac{(\dot{W}_{thp} + \dot{W}_{tlp} + \dot{W}_{cp}) / \dot{m}_1}{\dot{Q} / \dot{m}_1}$$

$$\dot{W}_{thp} = \dot{m}_1 w_{thp_a}$$
$$\dot{W}_{thp} / \dot{m}_1 = w_{thp_a}$$
$$\dot{W}_{thp} / \dot{m}_1 = 146.6 \text{Btu / lbm}$$

$$\dot{W}_{tlp} = (\dot{m}_1 - \dot{m}_2) w_{tlp_a}$$
$$\dot{W}_{tlp} / \dot{m}_1 = (1 - \dot{m}_2 / \dot{m}_1) w_{tlp_a}$$
$$\dot{W}_{tlp} / \dot{m}_1 = (1 - 0.1585)(248.3)$$
$$\dot{W}_{tlp} / \dot{m}_1 = 208.94 \text{ Btu / lbm}$$

$$\dot{W}_{cp} = \dot{m}_1 w_{cp_a}$$
$$\dot{W}_{cp} / \dot{m}_1 = w_{cp_a}$$
$$\dot{W}_{cp} / \dot{m}_1 = -3.51 \text{Btu / lbm}$$

$$\dot{Q} = \dot{m}_1 q$$
$$\dot{Q} / \dot{m}_1 = q$$
$$\dot{Q} / \dot{m}_1 = 1084.7 \text{ Btu / lbm}$$

$$\eta = \frac{146.6 + 208.94 - 3.51}{1084.7} = 32.45\%$$

Example:

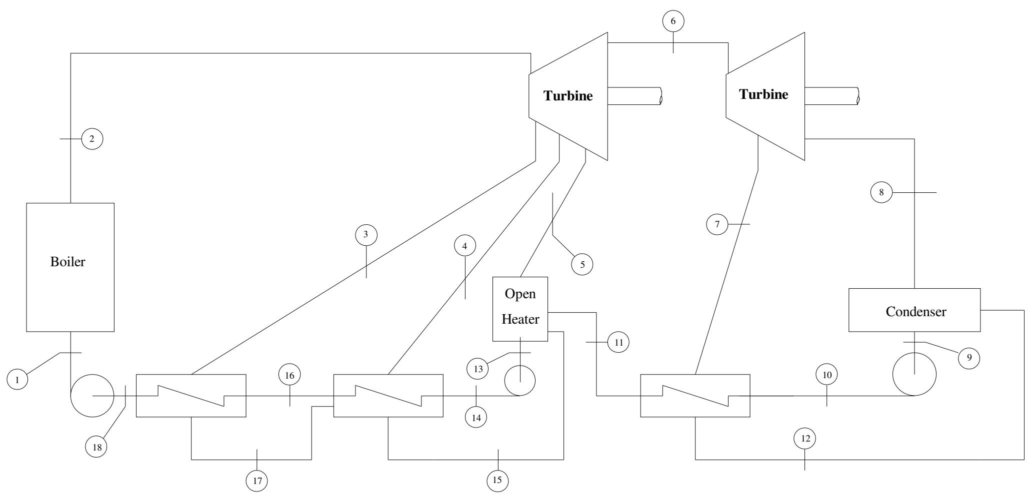
A steam power plant based on the regenerative cycle is illustrated below. Assuming the high pressure turbine is 90 % efficient and the low pressure turbine is 85 % efficient:

a) Determine the cycle efficiency. You may assume the condensate and low pressure feed pumps to be 90 % efficient.

b) Determine the temperature change across each of the feed water heaters.

c) Determine the power output of the turbines.

d) Determine the efficiency of the high pressure boiler feed pump.



Point	Pressure (psia)	Temperature (F)	Mass Flow Rate (lbm/hr)
1	1850	416	700,000
2	1265	925	
3	330		61,000
4	130		61,000
5	48.5		24,000
6	20		
7	11		54,000
8	0.75		
9			
10			
11			
12			
13			
14			
15			
16			
17			
18	400		

SOLUTION

High Pressure Turbine

h_2 = 1428.75 Btu/lbm
s_2 = 1.5851

High Pressure Tap

At P_3 = 330 psia and an entropy of 1.5851, the steam is superheated. From the superheat tables

h_{3s} = 1281.93 Btu/lbm

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - \eta_t (h_2 - h_{3s})$$

h_3 = 1428.75 - (0.90)(1428.75 - 1281.93) = 1296.61 Btu/lbm

Intermediate Pressure Tap

At P_4 = 130 psia and an entropy of 1.5851, the steam is superheated. From the superheat tables

h_{4s} = 1194.9 Btu/lbm

$$\eta_t = \frac{h_2 - h_4}{h_2 - h_{4s}} \Rightarrow h_4 = h_2 - \eta_t (h_2 - h_{4s})$$

h_4 = 1428.75 - (0.90)(1428.75 - 1194.9) = 1218.29 Btu/lbm

Low Pressure Tap

$$x_{5s} = \frac{s_2 - s_f}{s_{fg}} \ @\ P_5 = 48.5\, psia$$

$$h_{5s} = h_f + x_{5s} h_{fg} \ @\ P_5 = 48.5\, psia$$

s_f = 0.4083
s_{fg} = 1.2530

h_f = 248.1
h_{fg} = 925.4

$$x_{5s} = \frac{1.5851 - 0.4083}{1.2530} = 0.9392$$

h_{5s} = 248.1 + (0.9392)(925.4) = 1117.2 Btu/lbm

$$\eta_t = \frac{h_2 - h_5}{h_2 - h_{5s}} \Rightarrow h_5 = h_2 - \eta_t (h_2 - h_{5s})$$

h_5 = 1428.75 - (0.9)(1428.75 - 1117.2) = 1148.4 Btu/lbm

High Pressure Turbine Exhaust

$$x_{6s} = \frac{s_6 - s_f}{s_{fg}} \text{ @ } P_6 = 20\, psia$$

$$h_{6s} = h_f + x_{6s} h_{fg} \text{ @ } P_6 = 20\, psia$$

s_f = 0.3358
s_{fg} = 1.3962

h_f = 196.27 Btu/lbm
h_{fg} = 960.1 Btu/lbm

$$x_{6s} = \frac{1.5851 - 0.3358}{1.3962} = 0.8948$$

h_{6s} = 196.27 + (0.8948)(960.1) = 1055.4 Btu/lbm

$$\eta_t = \frac{h_2 - h_6}{h_2 - h_{6s}} \Rightarrow h_6 = h_2 - \eta_t (h_2 - h_{6s})$$

h_6 = 1428.75 - (0.9)(1428.75 - 1055.4) = 1092.7 Btu/lbm

Low Pressure Turbine

h_6 = 1092.7 Btu/lbm
$s_6 = s_f + x_{6a} s_{fg}$

$$x_{6a} = \frac{h_6 - h_f}{h_{fg}} = \frac{1092.7 - 196.27}{960.1} = 0.9337$$

s_6 = 0.3358 + (0.9337)(1.3962) = 1.6394

Low Pressure Tap

$$x_{7s} = \frac{s_6 - s_f}{s_{fg}} \text{ @ } P_7 = 11\, psia$$

$$h_{7s} = h_f + x_{7s} h_{fg} \text{ @ } P_7 = 11\, psia$$

s_f = 0.2896
s_{fg} = 1.4917

h_f = 165.25 Btu/lbm
h_{fg} = 979.62 Btu/lbm

$$x_{7s} = \frac{1.6394 - 0.2896}{1.4917} = 0.9049$$

h_{7s} = 165.25 + (0.9049)(979.62) = 1051.71 Btu/lbm

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} \Rightarrow h_7 = h_6 - \eta_t (h_6 - h_{7s})$$

h_7 = 1092.7 - (0.85)(1092.7 - 1051.71) = 1057.9 Btu/lbm

Low Pressure Turbine Exhaust

$$x_{8s} = \frac{s_6 - s_f}{s_{fg}} \text{ @ } P_8 = 0.75\, psia$$

$$h_{8s} = h_f + x_{8s} h_{fg} \text{ @ } P_8 = 0.75\, psia$$

s_f = 0.1126
s_{fg} = 1.895

h_f = 58.68 Btu/lbm
h_{fg} = 1042.4 Btu/lbm

$$x_{8s} = \frac{1.6394 - 0.1126}{1.895} = 0.8057$$

h_{8s} = 58.68 + (0.8057)(1042.4) = 898.5 Btu/lbm

$$\eta_t = \frac{h_6 - h_8}{h_6 - h_{8s}} \Rightarrow h_8 = h_6 - \eta_t (h_6 - h_{8s})$$

h_8 = 1092.7 - (0.85)(1092.7 - 898.5) = 927.63 Btu/lbm

Condensate Pump

$$-w_{cps} = \upsilon(P_{10} - P_9) = (0.0161)(48.5 - 0.75)(144/778) = 0.142 \text{ Btu/lbm}$$

$$-w_{cp} = \frac{-w_{cps}}{\eta_p} = \frac{0.142}{0.90} = 0.158 \text{ Btu/lbm}$$

$$h_{10} = h_9 - w_{cp}$$

h_{10} = 58.68 + 0.158 = 58.84 Btu/lbm

Low Pressure Feed Heater

$$\dot{m}_7 h_7 + \dot{m}_{10} h_{10} = \dot{m}_7 h_{12} + \dot{m}_{10} h_{11}$$

Since all the mass flow rates are known, we can solve for the fluid enthalpy leaving the heater as

$$h_{11} = h_{10} + \frac{\dot{m}_7}{\dot{m}_{10}}(h_7 - h_{12})$$

$$\dot{m}_{10} = \dot{m}_1 - \dot{m}_3 - \dot{m}_4 - \dot{m}_5$$
$$= 700{,}000 - 61{,}000 - 61{,}000 - 24{,}000$$
$$\dot{m}_{10} = 554{,}000 \text{ lbm/hr}$$

$h_{12} = h_f$ @ P_7 = 11 psia = 165.25 Btu/lbm

h_{11} = 58.84 + (54/554)(1057.9 - 165.25) = 145.85 Btu/lbm

<u>Open Feed Heater</u>

$$\dot{m}_5 h_5 + \dot{m}_{10} h_{11} + (\dot{m}_3 + \dot{m}_4) h_{15} = \dot{m}_1 h_{13}$$

Again the mass flow rates are known, such that the enthalpy leaving the open heater is

$$h_{13} = \frac{\dot{m}_5 h_5 + \dot{m}_{10} h_{11} + (\dot{m}_3 + \dot{m}_4) h_{15}}{\dot{m}_1}$$

h_5 = 1148.4 Btu/lbm
h_{11} = 145.85 Btu/lbm
$h_{15} = h_f$ @ P_4 = 130 psia = 319.0 Btu/lbm

h_{13} = (24/700)(1148.4)+(554/700)(45.85)+(122/700)(319) = 210.4 Btu/lbm

<u>Intermediate Pressure Pump</u>

$$-w_{ips} = \upsilon(P_{14} - P_{13})$$

The specific volume is taken as that of a saturated liquid corresponding to a liquid enthalpy of h_{13} = 210.4 Btu/lbm.

υ = 0.0169 ft^3/lbm

$$-w_{ips} = (0.0169)(400 - 48.5)(144/778) = 1.10 \text{ Btu/lbm}$$

$$-w_{ip} = \frac{-w_{ips}}{\eta_p} = \frac{1.1}{0.9} = 1.22 \text{ Btu/lbm}$$

$$h_{14} = h_{13} - w_{ip}$$

h_{14} = 210.4 + 1.22 = 211.6 Btu/lbm

<u>Intermediate Pressure Feed Heater</u>

$$\dot{m}_4 h_4 + \dot{m}_3 h_{17} + \dot{m}_1 h_{14} = \dot{m}_1 h_{16} + (\dot{m}_3 + \dot{m}_4) h_{15}$$

Solving for h_{16} gives

$$h_{16} = h_{14} + \frac{\dot{m}_4}{\dot{m}_1} h_4 + \frac{\dot{m}_3}{\dot{m}_1} h_{17} - \frac{(\dot{m}_3 + \dot{m}_4)}{\dot{m}_1} h_{15}$$

h_4 = 1218.29 Btu/lbm
h_{14} = 211.6 Btu/lbm
$h_{15} = h_f$ @ P_4 = 130 psia = 319.0 Btu/lbm
$h_{17} = h_f$ @ P_3 = 330 psia = 403.48 Btu/lbm

h_{16} = 211.6 + (61/700)(1218.29)+(61/700)(403.48)-(122/700)(319) = 297.3 Btu/lbm

High Pressure Heater

$$\dot{m}_1 h_{16} + \dot{m}_3 h_3 = \dot{m}_1 h_{18} + \dot{m}_3 h_{17}$$

Solving for h_{18} gives

$$h_{18} = h_{16} + \frac{\dot{m}_3}{\dot{m}_1}(h_3 - h_{17})$$

h_3 = 1296.61 Btu/lbm
h_{16} = 297.3 Btu/lbm
h_{17} = 403.48 Btu/lbm

h_{18} = 297.3 + (61/700)(1296.61 - 403.48) = 375.1 Btu/lbm

High Pressure Boiler Feed Pump

$$-w_{hps} = \upsilon(P_1 - P_{18})$$

The specific volume is taken as that of a saturated liquid corresponding to a liquid enthalpy of h_{18} = 375.1 Btu/lbm.

υ = 0.0186 ft^3/lbm

$$-w_{hps} = (0.0186)(1850 - 400)(144/778) = 4.99 \text{ Btu/lbm}$$

$$\eta_p = \frac{-w_{hps}}{-w_{hp}}$$

$$-w_{hp} = h_1 - h_{18}$$

$h_1 \cong$ 392.5 Btu/lbm
h_{18} = 375.1 Btu/lbm

$$-w_{hp} = 392.5 - 375.1 = 17.4 \text{ Btu/lbm}$$

$$\eta_p = \frac{4.99}{17.4} = 28.73\ \%$$

<u>Boiler</u>

$q = h_2 - h_1$ = 1428.75 - 392.5 = 1036.25 Btu/lbm

<u>Cycle Efficiency</u>

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}} = \frac{\dot{W}_{thp} + \dot{W}_{tlp} + \dot{W}_{cp} + \dot{W}_{ip} + \dot{W}_{hp}}{\dot{Q}}$$

$$\begin{aligned}\dot{W}_{thp} &= \dot{m}_1 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4 - \dot{m}_5 h_5 - (\dot{m}_1 - \dot{m}_3 - \dot{m}_4 - \dot{m}_5) h_6 \\ &= (700{,}000)(1428.75) - (61{,}000)(1296.61) - (61{,}000)(1218.29) - (24{,}000)(1148.4) - (554{,}000)(1092.7) \\ &= 2.138 \times 10^8 \text{ Btu/hr}\end{aligned}$$

$$\begin{aligned}\dot{W}_{tlp} &= \dot{m}_6 h_6 - \dot{m}_7 h_7 - (\dot{m}_6 - \dot{m}_7) h_8 \\ &= (554{,}000)(1092.7) - (54{,}000)(1057.9) - (500{,}000)(927.63) \\ &= 8.44 \times 10^7 \text{ Btu/hr}\end{aligned}$$

$$\begin{aligned}\dot{W}_{cp} &= \dot{m}_{10} w_{cp} \\ &= (554{,}000)(-0.158) \\ &= -87532 \text{ Btu/hr}\end{aligned}$$

$$\begin{aligned}\dot{W}_{ip} &= \dot{m}_1 w_{ip} \\ &= (700{,}000)(-1.22) \\ &= -854{,}000 \text{ Btu/hr}\end{aligned}$$

$$\begin{aligned}\dot{W}_{hp} &= \dot{m}_1 w_{hp} \\ &= (700{,}000)(-17.4) \\ &= -1.218 \times 10^7 \text{Btu/hr}\end{aligned}$$

$$\begin{aligned}\dot{Q} &= \dot{m}_1 q \\ &= (700{,}000)(1036.25) \\ &= 7.254 \times 10^8 \text{ Btu/hr}\end{aligned}$$

$$\begin{aligned}\eta &= \frac{2.13 \times 10^8 + 8.44 \times 10^7 - 87{,}532 - 854{,}000 - 1.218 \times 10^7}{7.254 \times 10^8} \\ &= 39.19\ \%\end{aligned}$$

The temperatures corresponding to the heater inlet and outlets are approximately

$T_{10} \cong 91$ F
$T_{11} \cong 178$ F

$T_{14} \cong 243$ F
$T_{16} \cong 327$ F
$T_{18} \cong 400$ F

The change in temperature across the individual heaters is then

1) Low pressure heater, $\Delta T = 178 - 91 = 87$ F
2) Intermediate pressure heater, $\Delta T = 327 - 243 = 84$ F
3) High pressure heater, $\Delta T = 400 - 327 = 73$ F